Mechanism Design

Reinterpreting Optimal Auction

2023-04-03

The Simple Economics of Optimal Auctions

- Journal of Political Economy (1989)

Optimal Auction Problem

- Maximize the seller's expected profit (Vickrey, 1961)
  Then... what is auction?

- Auction setting
  - Seller values an item at 0
  - n risk-neutral, symmetric bidders
  - Each bidder knows only their own value v (sealed-bid)
  - First-price: highest bidder wins and pays his value
  - Second-price: highest bidder wins and pays the second-highest value

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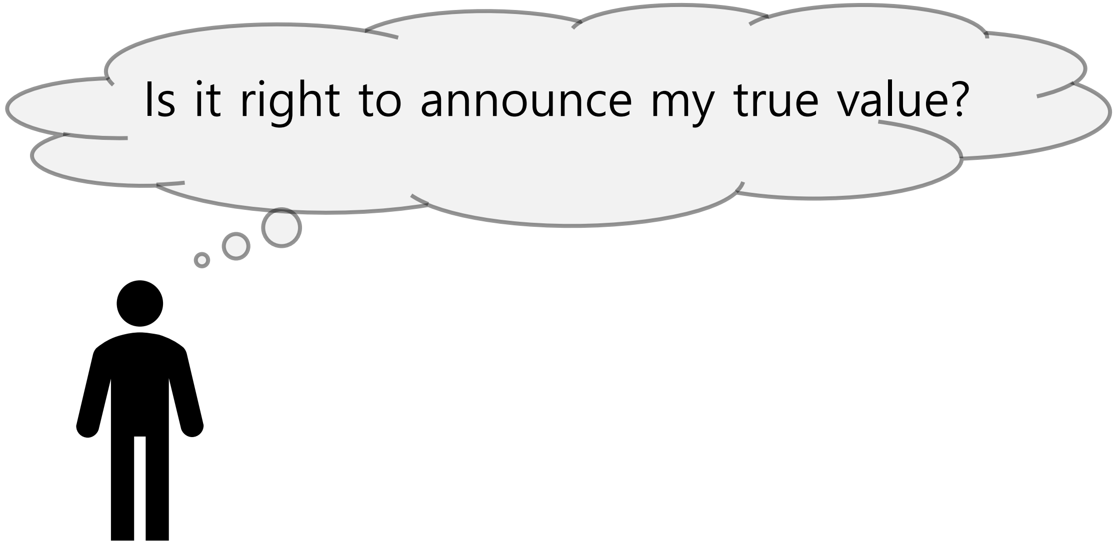
Optimal Auction Problem: second-price



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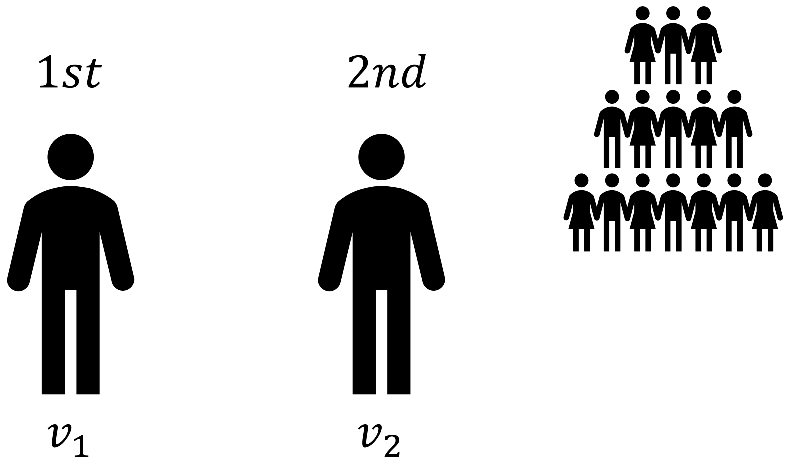
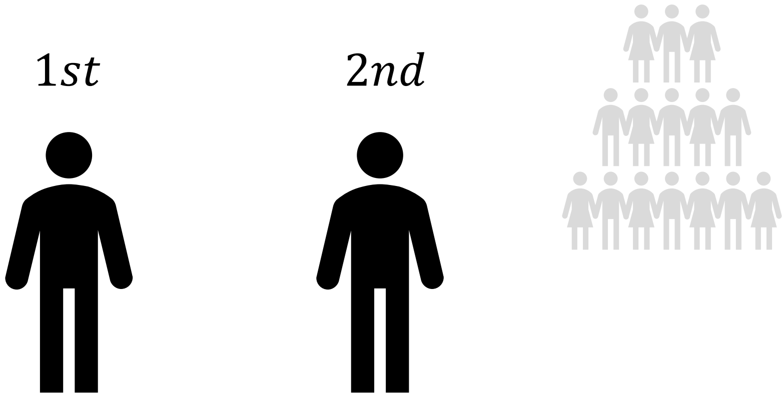


Optimal Auction Problem: second-price

$1st$ $2nd$



v_1 v_2

Optimal Auction Problem: second-price

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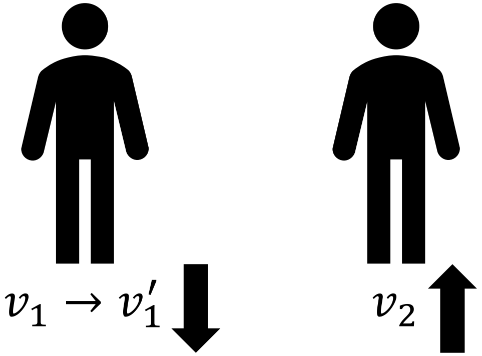
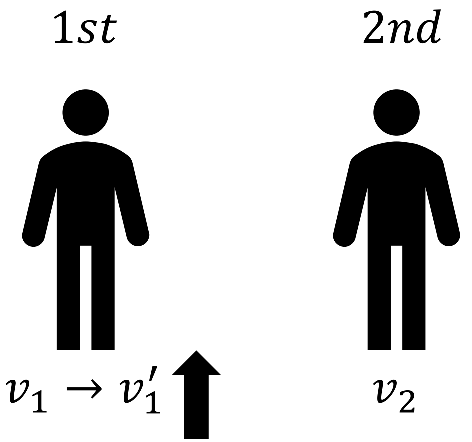


$v_1 \rightarrow v_1'$ v_2

Underbid: not to be the high bidder although his value is the highest

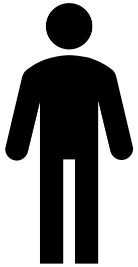
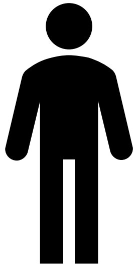
Optimal Auction Problem: second-price



Bid more: pay more than his value

Optimal Auction Problem: second-price

$1st$ $2nd$

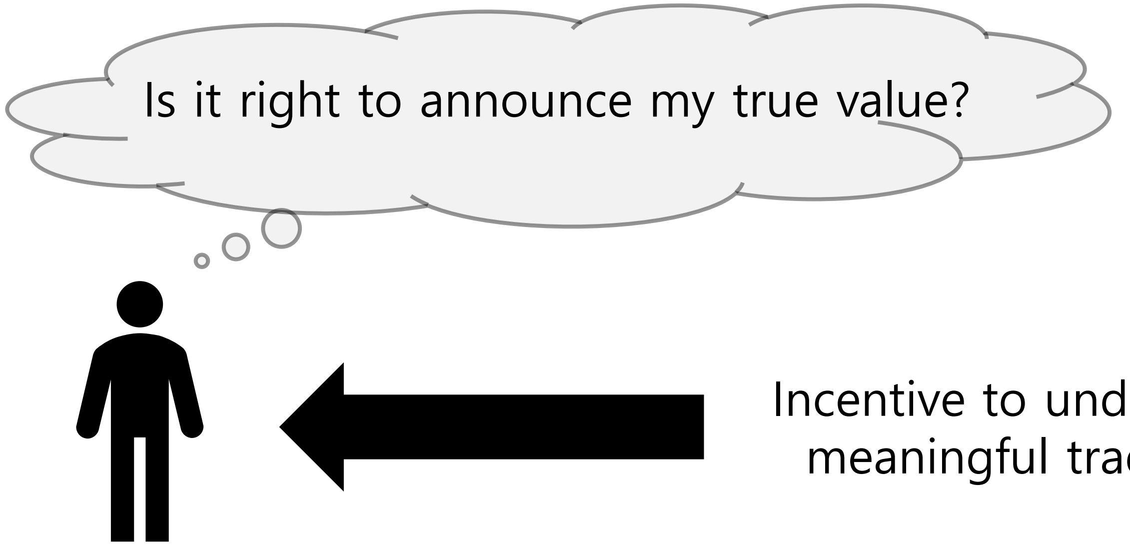
v_1 v_2

Underbid: not to be the high bidder although his value is the highest

Bid more: pay more than his value

→ Dominant strategy of bidding his <u>true value</u>

Optimal Auction Problem: first-price



Incentive to underbid:
meaningful trade-off against prob. to win & lower payment

For the rest of the section, we assume all the auctions are second-price!

Optimal Auction Problem

Extension (Myerson, 1981)

- Asymmetric bidders: probability distribution of values to be common knowledge
- All possible ways of selling the goods

  ascending, descending oral auctions, all-pay auctions, and many more…

→ Simplification!!

insights on revelation principle

Optimal Auction Problem

Direct revelation mechanism

- ensure buyers are willing to participate
- ensure buyers to announce his true valuation

→ Simple constrained maximization problem

Optimal Auction Problem

Simple constrained maximization problem

- Maximize the seller’s expected revenues
- Participation constraint: bidder receive non-negative expected surplus
- Incentive constraint: reveal their true valuations

Construction of Optimal Auction

Maximize the seller's expected revenue

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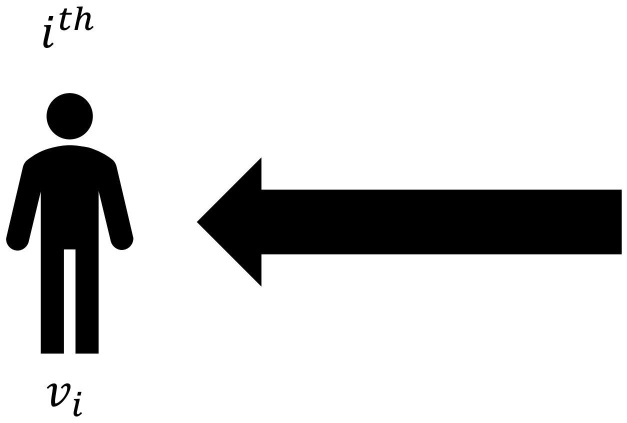
Maximize the seller's expected revenue



Drawn from F_i with $F_i(\underline{v}_i) = 0$ and $F_i(\overline{v}_i) = 1$

Construction of Optimal Auction

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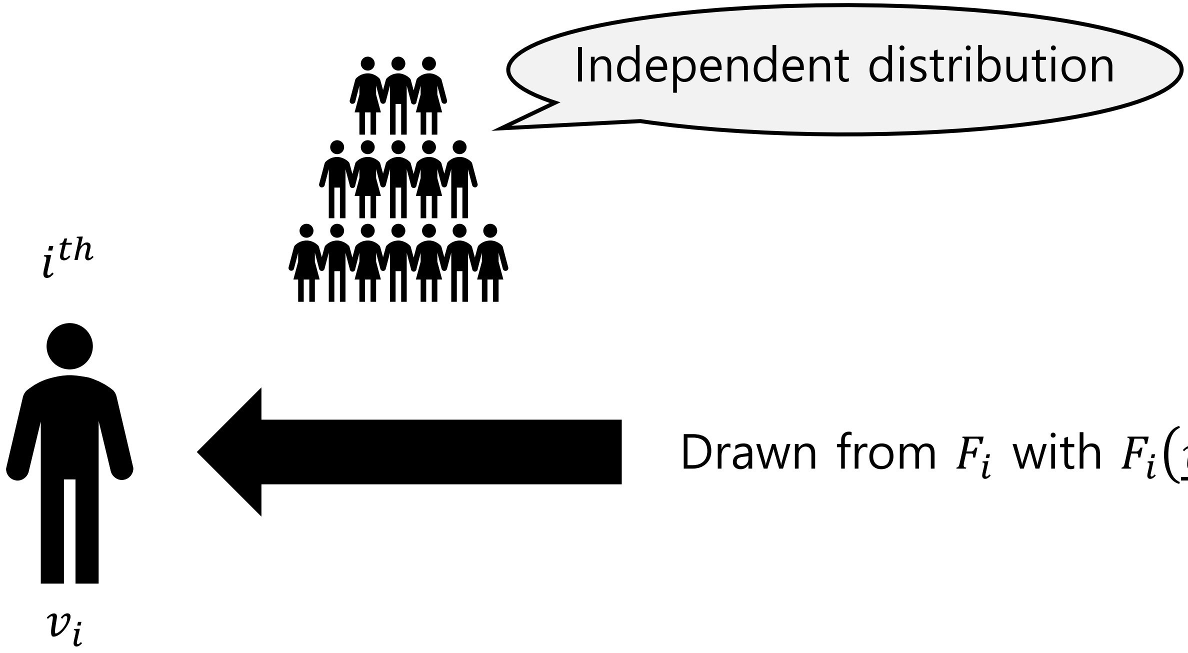

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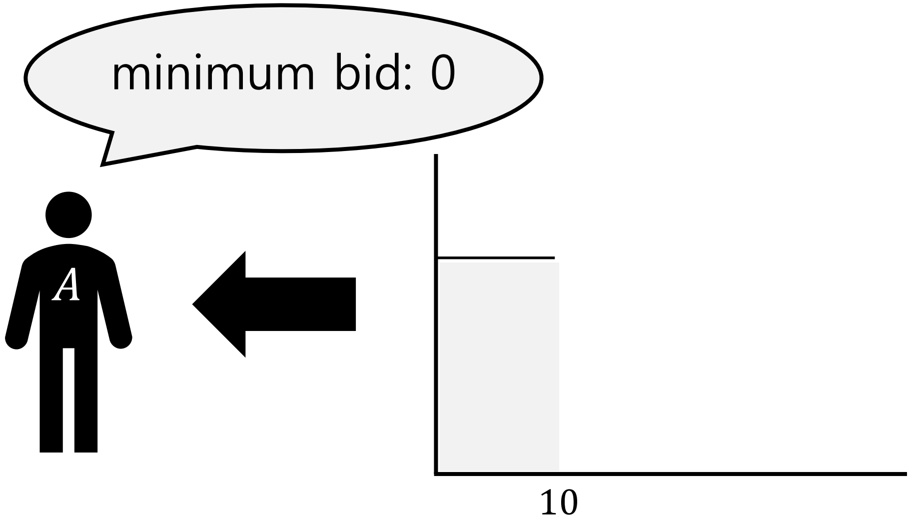

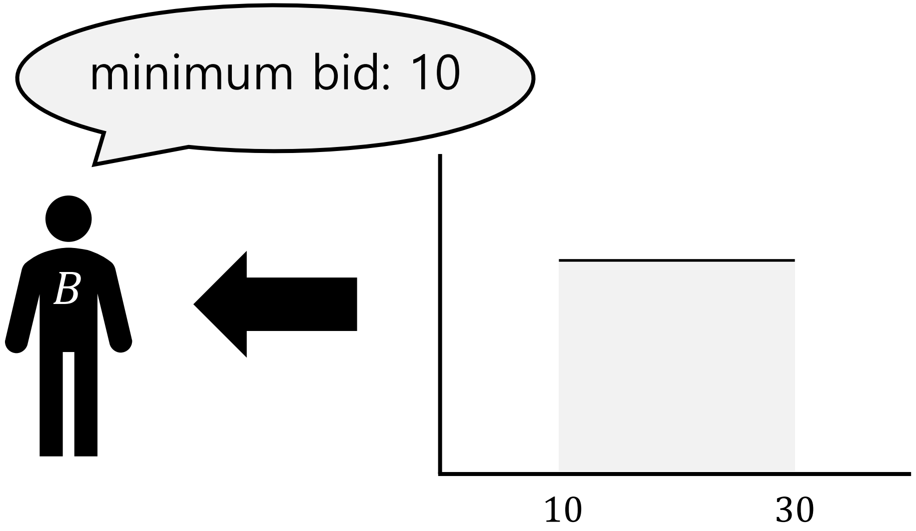

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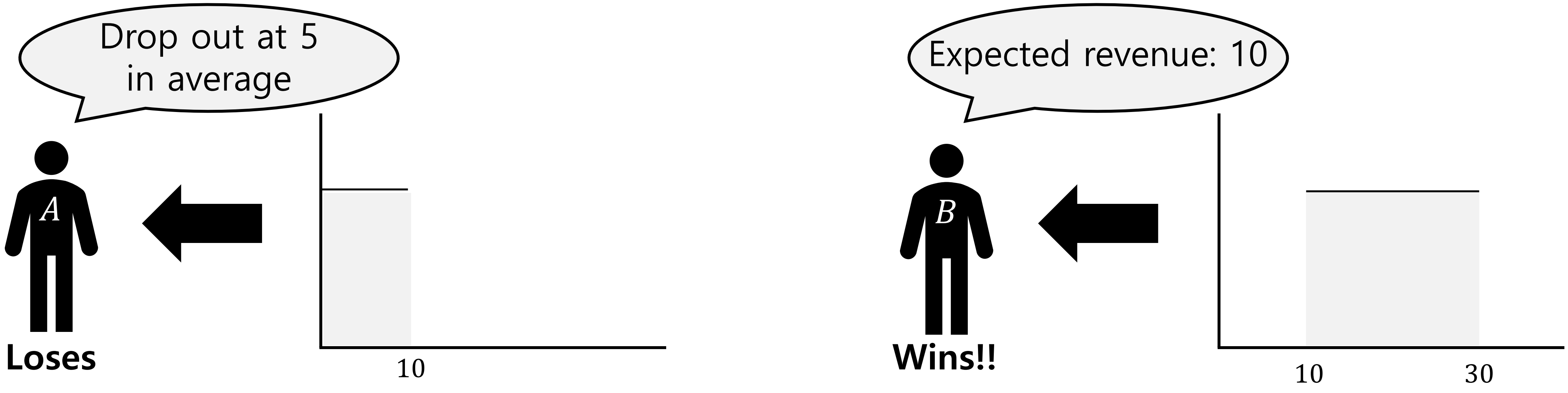
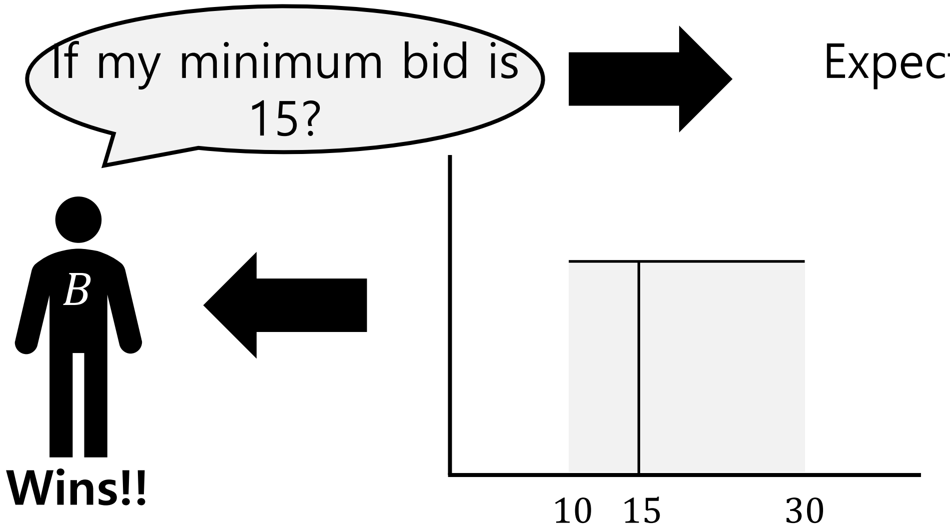


Construction of Optimal Auction

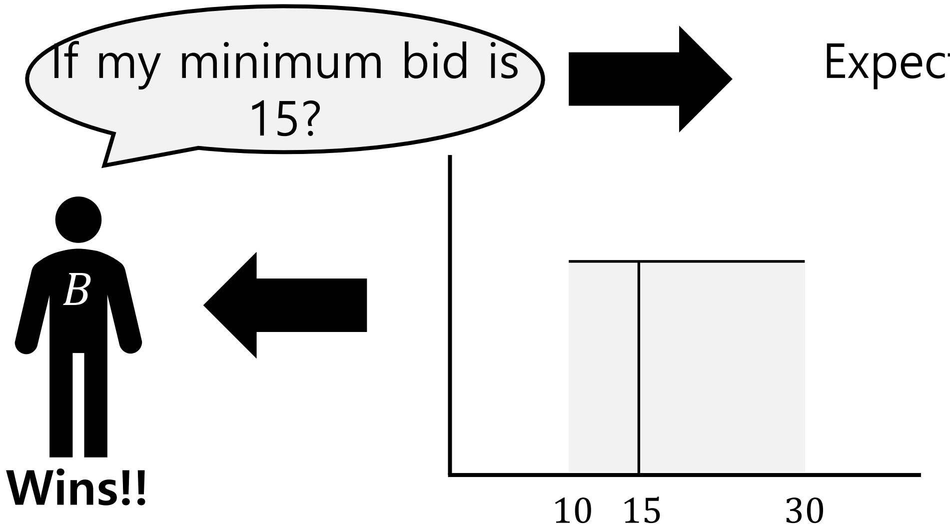
Maximize the seller’s expected revenue



Expected revenue: 11.25

Construction of Optimal Auction

Maximize the seller's expected revenue



Expected revenue: 11.25

How to construct a mechanism with maximum possible expected revenue?

Construction of Optimal Auction

- For each bidder, graph the inverse of his F_i function
  - Value v on the Y-axis (price)
  - Prob. $q \equiv 1 - F_i(v)$ on the X-axis (quiantity)

→ Demand curve

Construction of Optimal Auction

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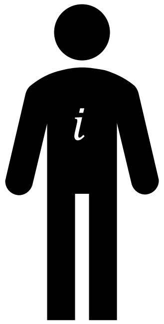

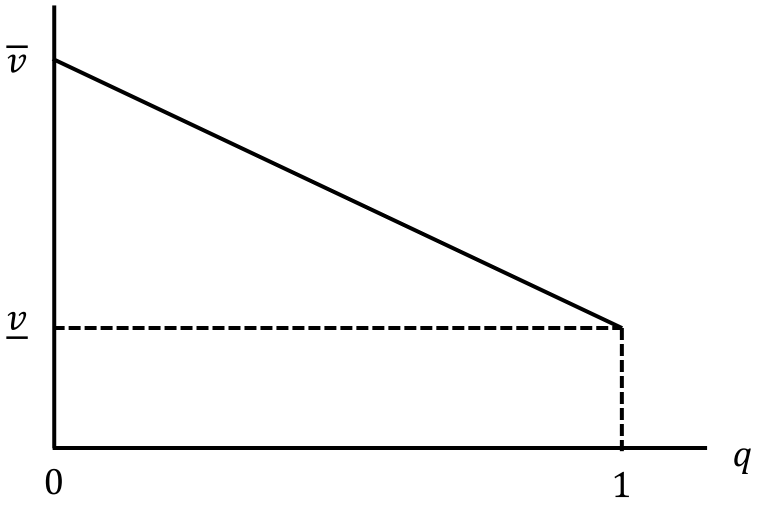

Construction of Optimal Auction

- Demand curve
  - Value v on the Y-axis (price)
  - Prob. $q \equiv 1 - F_i(v)$ on the X-axis (quiantity)

- Marginal revenue curve
  - Multiply quantity times price
  - $q = 1 - F_i(v)$
  - $v = F_i^{-1}(1 - q)$

$$MR_i(v) = v - \frac{1 - F_i(v)}{f_i(v)}$$

$*$ Recall that we assumed that marginal revenue is monotonic (downward sloping)

Construction of Optimal Auction

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Construction of Optimal Auction

- Demand curve & marginal revenue curve
- Second marginal revenue auction

  1) each bidder announces his value

Construction of Optimal Auction

- Demand curve & marginal revenue curve
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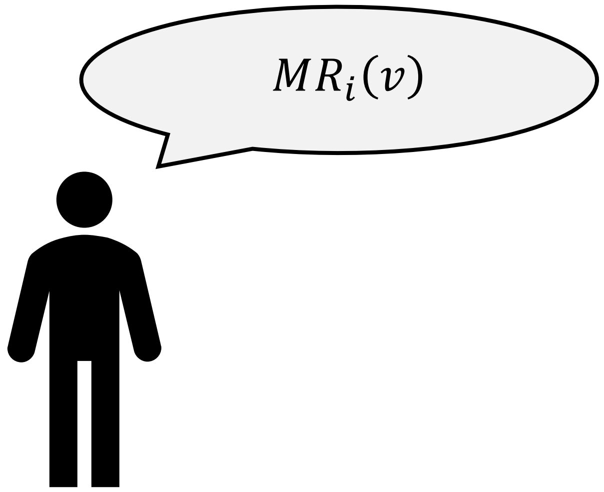
  2) value → marginal revenue



Construction of Optimal Auction

- Demand curve & marginal revenue curve
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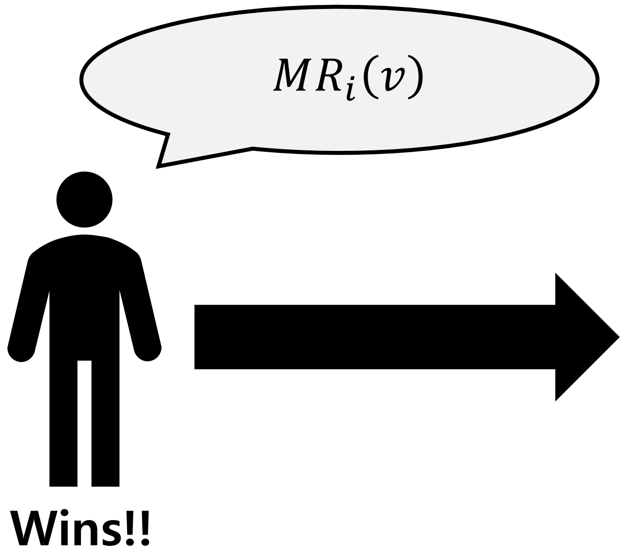
  3) Bidder with the highest marginal revenue wins!



lowest value the bidder can announce without losing ($MR_i(v) = 0$)

Construction of Optimal Auction

- Demand curve & marginal revenue curve
- Second marginal revenue auction

  3′) No bidder with a non-negative marginal revenue

Construction of Optimal Auction

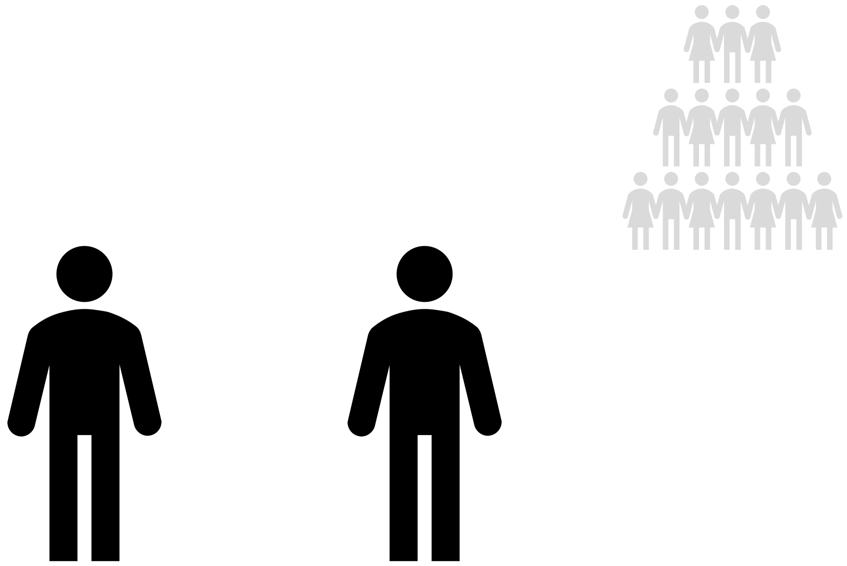
- Demand curve & marginal revenue curve
- Second marginal revenue auction
  - ∗ Auction with only one bidder

Wins!!

Price equal to the value v_1^* of $MR_1(v_1^*) = 0$

Construction of Optimal Auction

- Demand curve & marginal revenue curve
- Second marginal revenue auction



Wins!!

Construction of Optimal Auction

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  - Symmetric bidders: equivalent to a second-price auction
  - Asymmetric bidders: not necessarily (highest price) = (highest marginal revenue)

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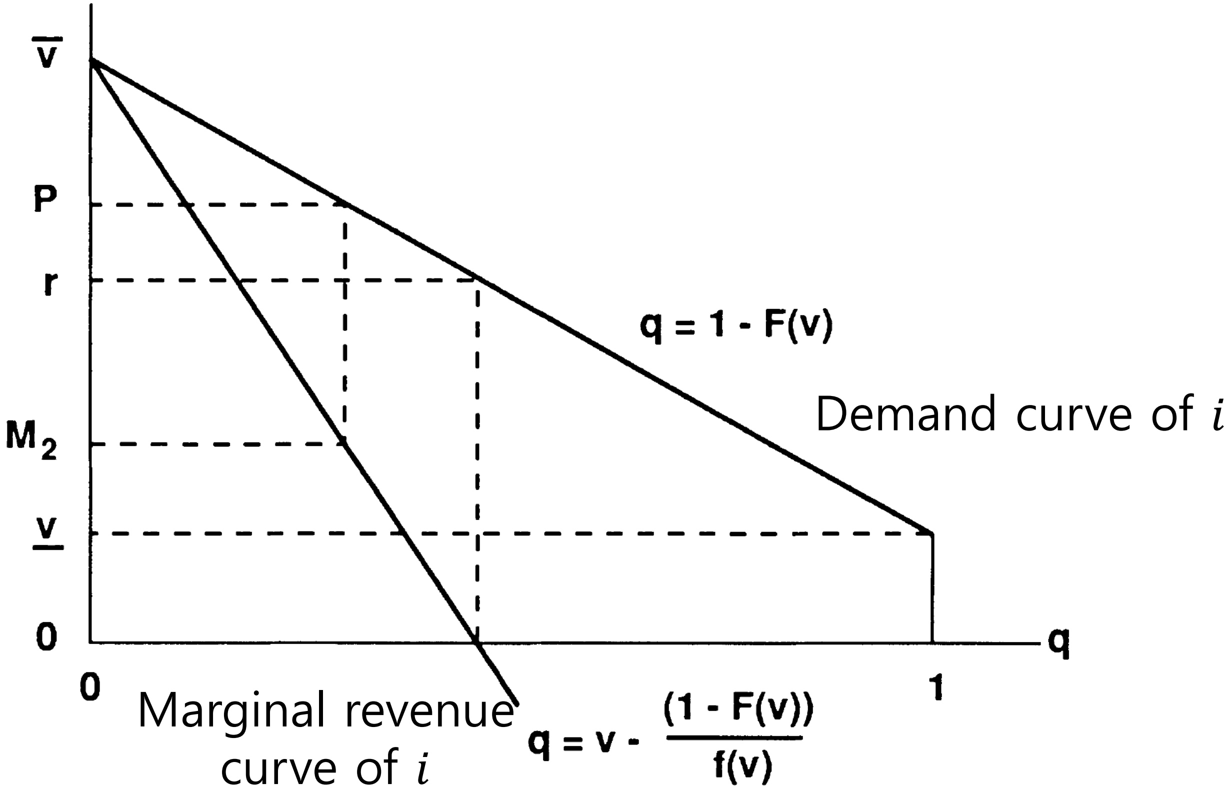
- How should I pay?

Construction of Optimal Auction

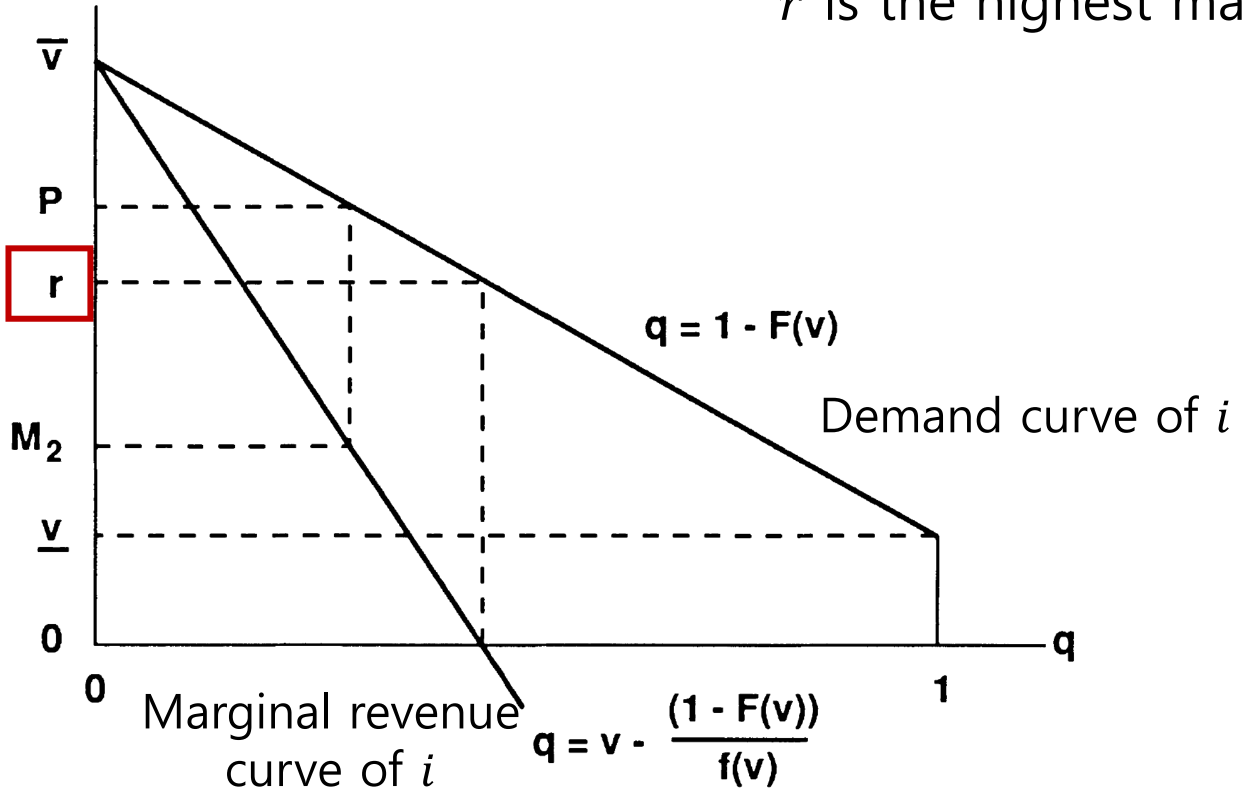
- Second marginal revenue auction



Construction of Optimal Auction

- Second marginal revenue auction

r is the highest marginal revenue the winning bidder i's price

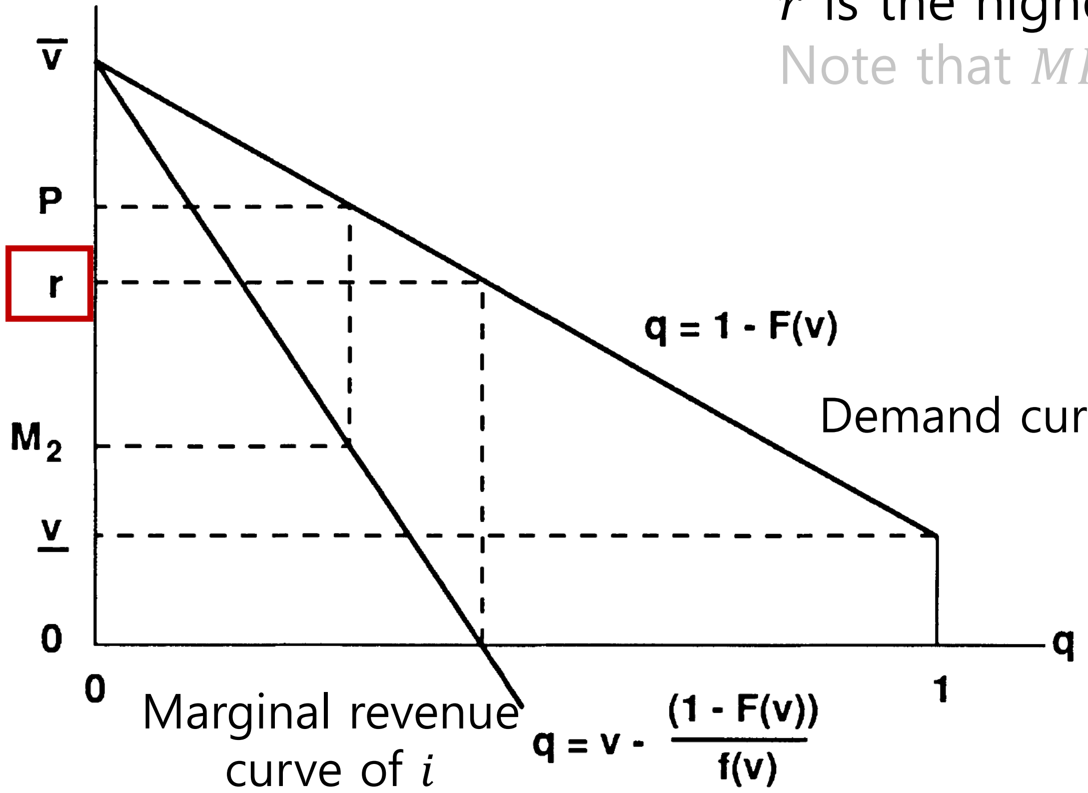


Construction of Optimal Auction

- Second marginal revenue auction

r is the highest marginal revenue the winning bidder i's price

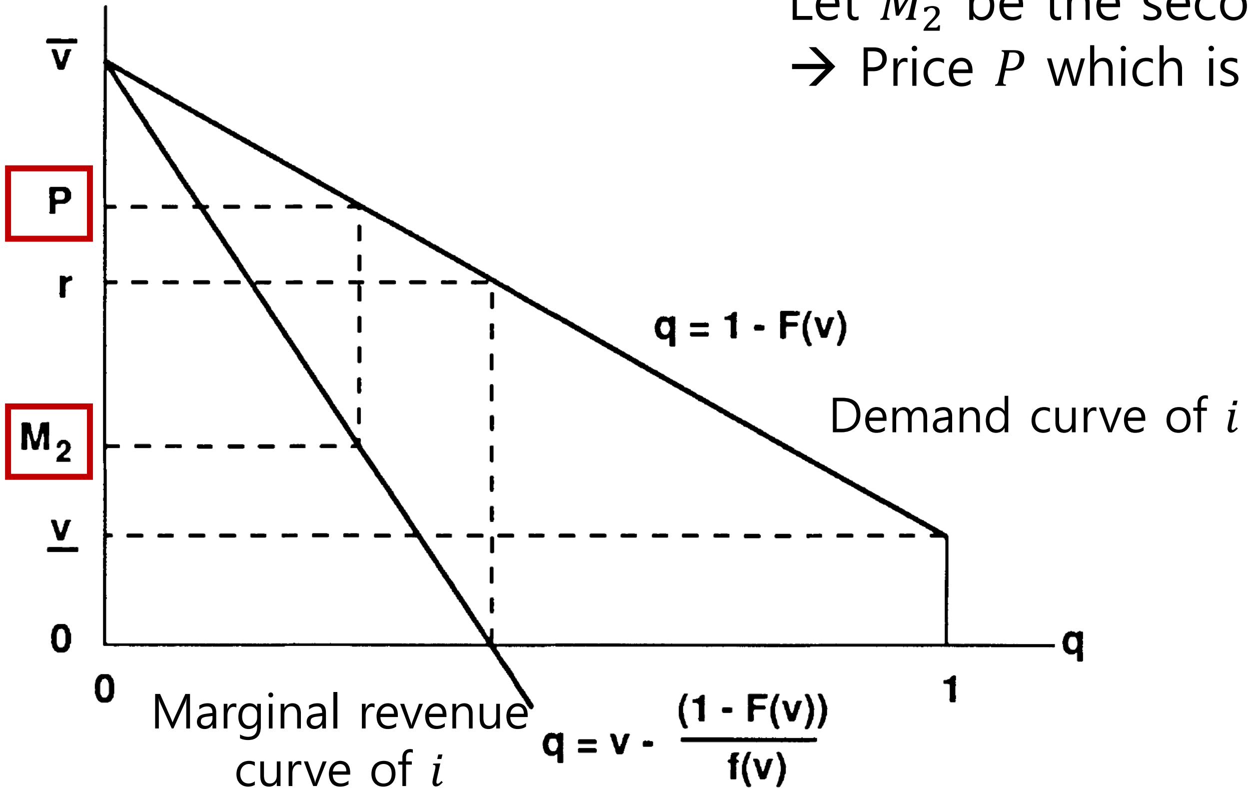
Note that $MR_i(r) = 0$!!!



Construction of Optimal Auction

- Second marginal revenue auction

Let M_2 be the second-highest marginal revenue
→ Price P which is $MR_i^{-1}(M_2)$



Construction of Optimal Auction

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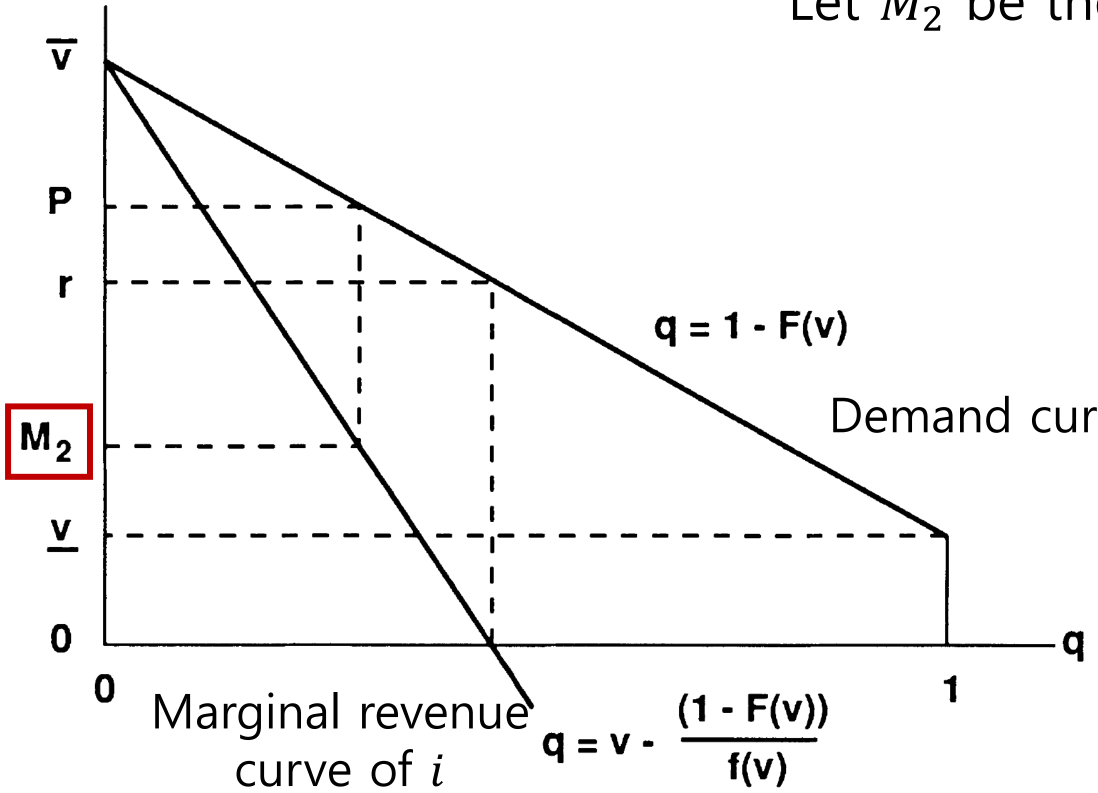

Demand curve of i

Marginal revenue curve of i

Construction of Optimal Auction

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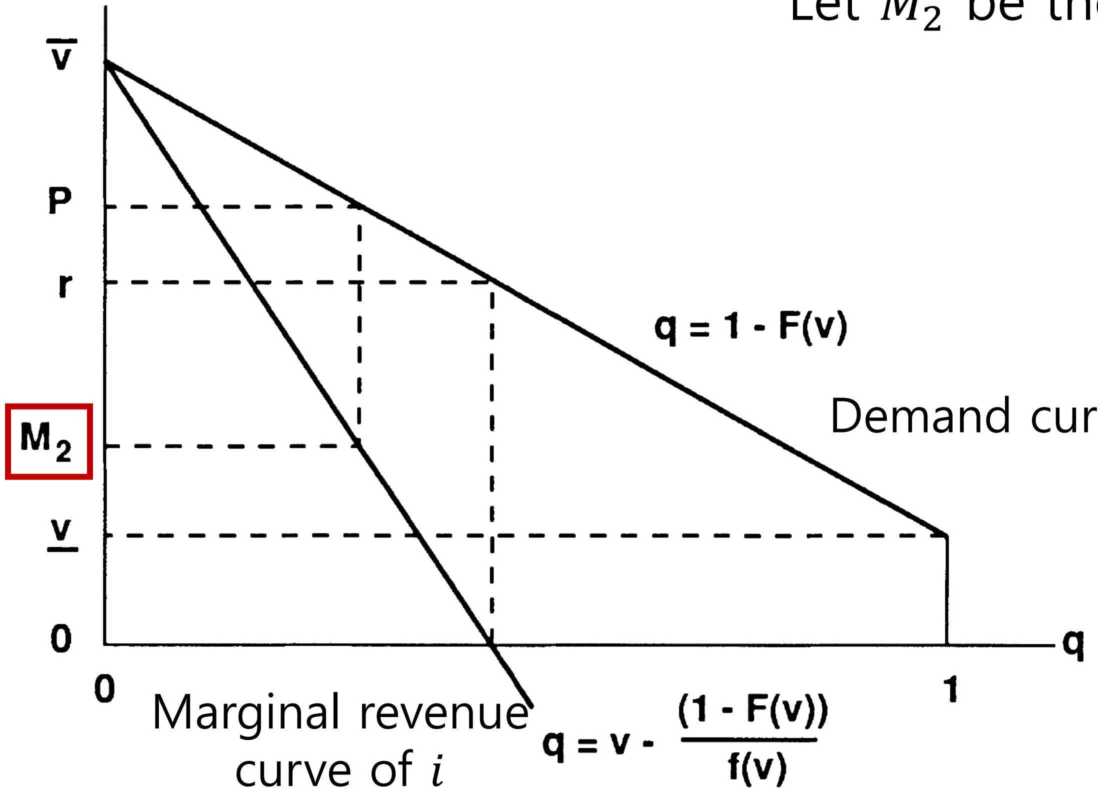

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- Pay $MR_i(M_2)$ where M_2 is the second-highest marginal revenue

Construction of Optimal Auction $*$

- Can generalize to an optimal mechanism of…
  - Seller sells k identical goods
  - Each buyer wants only one unit

→ $(k+1)$st marginal revenue auction

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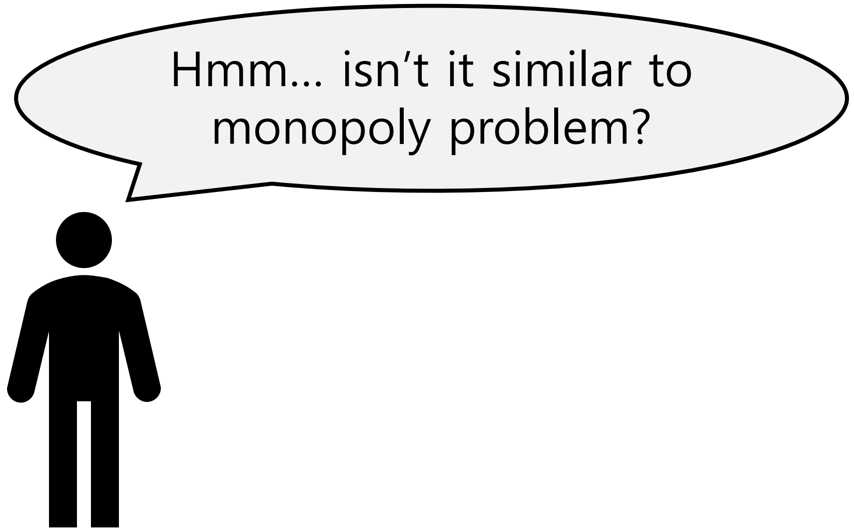
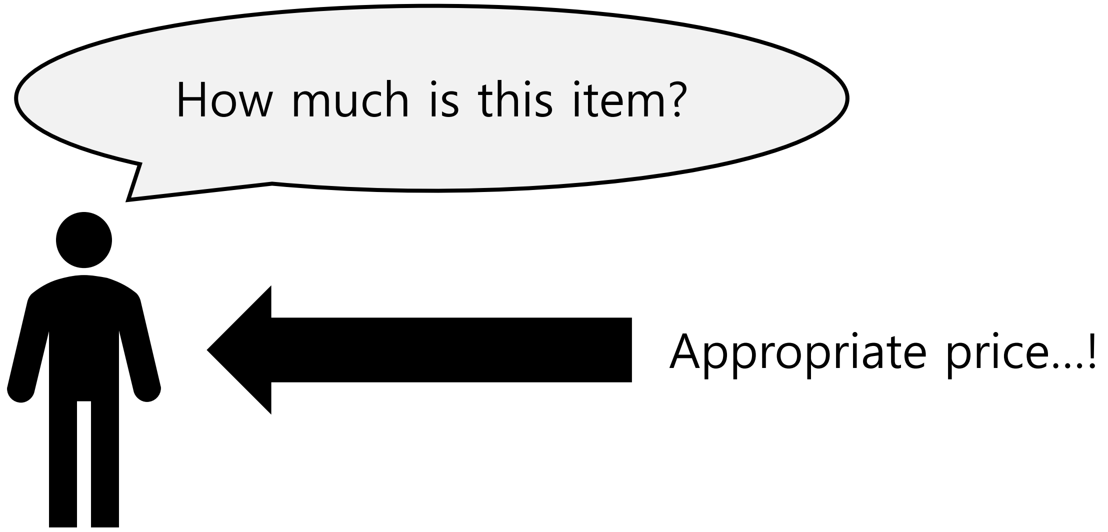


Price Discriminating Monopoly

- Third-degree monopoly price discrimination problem
  - Monopolist who sells in n different markets
  - Monopolist of capacity of $\tilde{Q}$ units (marginal cost 0 up to $\tilde{Q}$ units)

    $\tilde{Q}$ is a random variable in $\left[\underline{Q}, \overline{Q}\right]$ that takes on the value Q with probability $h(Q)$
  - Each customer (=buyer) buys at most one unit
  - Customers in market i have values in $[\underline{v}_i, \overline{v}_i]$ (values as <u>common knowledge</u>)

    $F_i(v)$ customers have a value $\leq v$



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    $F_i(v)$ customers have a value $\leq v$

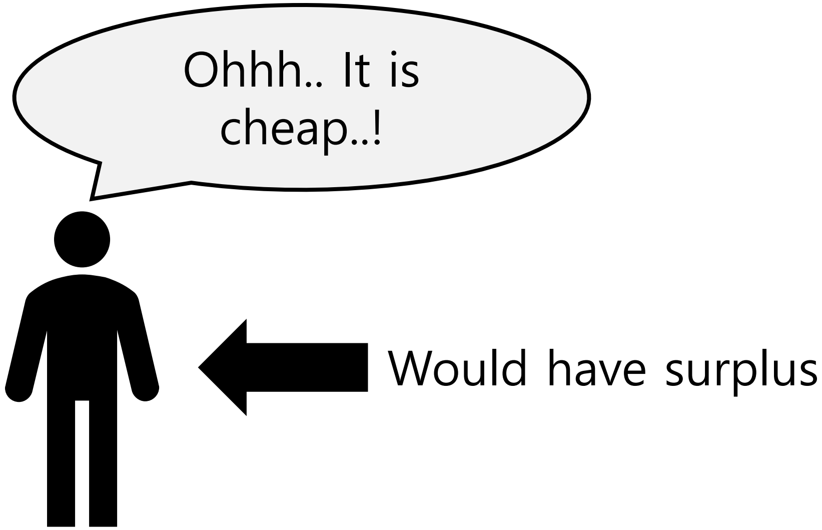
→ Maximize her expected profit by choosing price in each market

$*$ Recall that we assumed that marginal revenue is monotonic (downward sloping)

Price Discriminating Monopoly

- Third-degree monopoly price discrimination problem
  - Price depends on the capacity Q
  - $p_i(v, Q) \coloneqq$ prob. that a buyer in market i with value v acquires a unit if the monopolist's capacity is Q

  - $p_i(v, Q) = \begin{cases} 1 & \text{the monopolist chooses a price} \leq v \\ 0 & \text{the monopolist chooses a price} > v \end{cases}$



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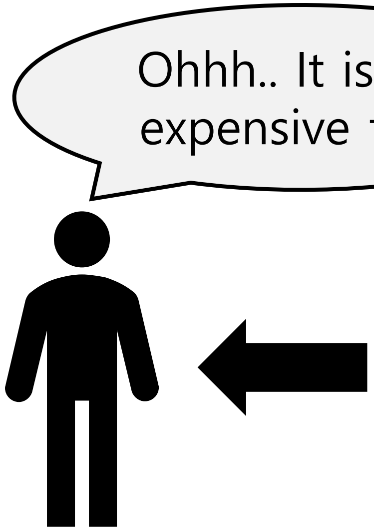

Would not buy

Price Discriminating Monopoly

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  - $p_i(v, Q) = \begin{cases} 1 & \text{the monopolist chooses a price} \leq v \\ 0 & \text{the monopolist chooses a price} > v \end{cases}$
  - Unconditional prob. of $p_i(v, Q)$

$$\overline{p}_i(v) \equiv \int_{\underline{Q}}^{\overline{Q}} p_i(v, Q) h(Q) dQ$$

Price Discriminating Monopoly

- Third-degree monopoly price discrimination problem

- The expected social value of the units sold:
  (buyers' expected consumer surplus)+(monopolist's expected revenues)

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- Maximize monopolist's expected revenue:
  ⬆(expected social values) – (expected consumer's surplus)⬇

- Expected social values

$$\int_{\underline{Q}}^{\overline{Q}} h(Q) \sum_{i=1}^{n} \int_{\underline{v}_i}^{\overline{v}_i} v f_i(v) p_i(v, Q) dv dQ$$

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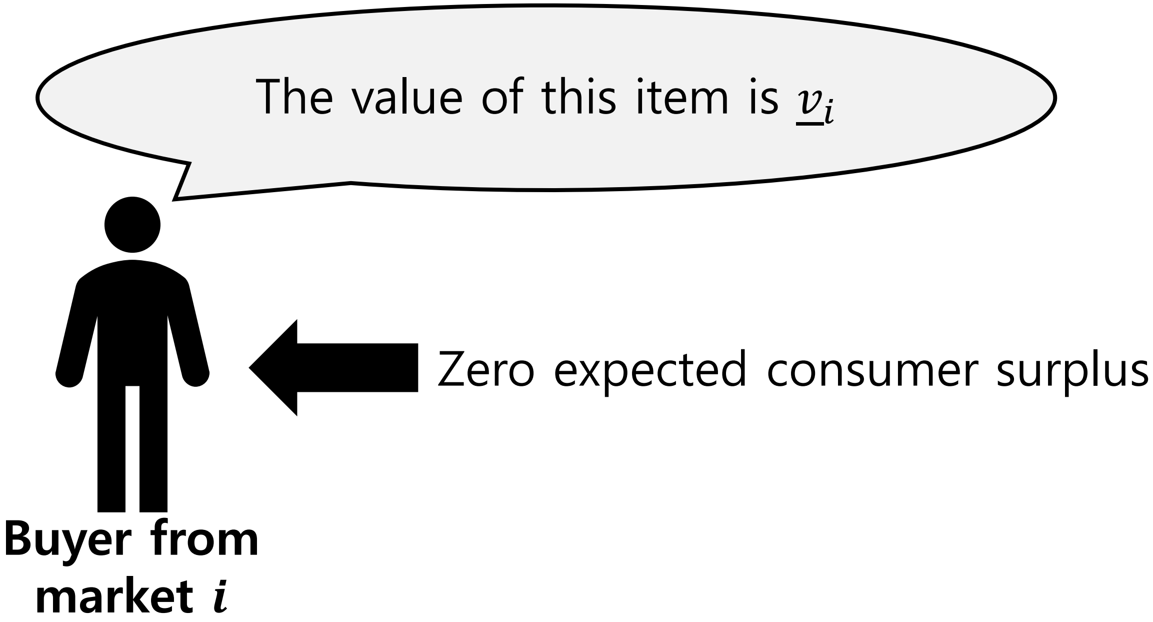

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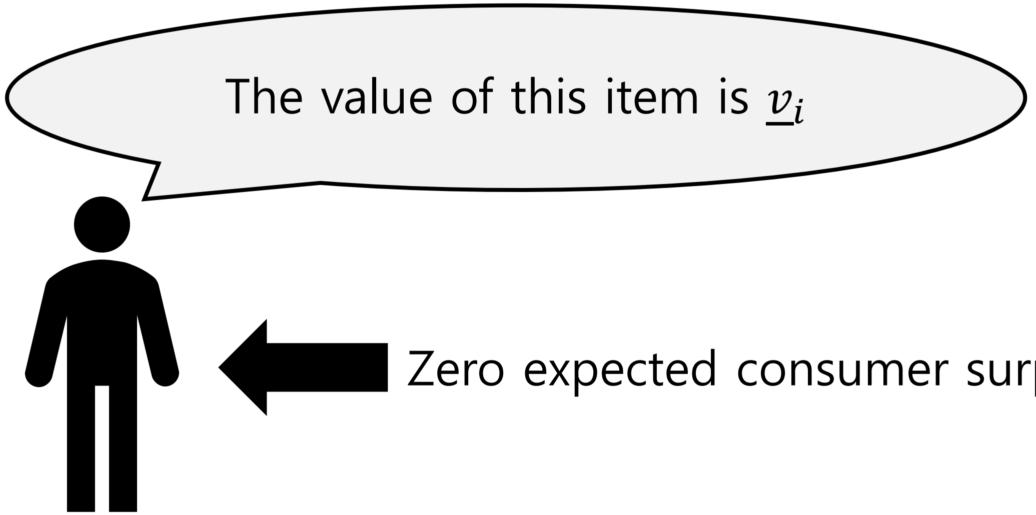


Zero expected consumer surplus

Buyer from market i

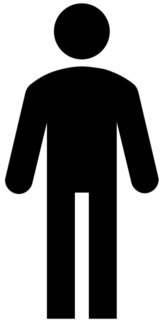
Price Discriminating Monopoly

- Maximize monopolist's expected revenue:
  ⬆(expected social values) – (expected consumer's surplus)⬇

  - Monopolist demand curve: $p_i = 100 - q_i$
  - Capacity: uniform distribution in $[0,100]$

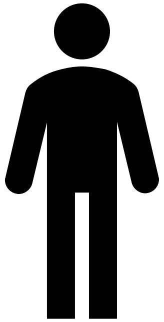
I will charge…
price 50 for 0.5 prob.
price $100x$ for for prob. $0.5 \le x \le 1$

The value of this item is 80..!



Buyer from market i

Wow..! Would have a positive expected consumer surplus



Monopolist

Price Discriminating Monopoly

- Maximize monopolist's expected revenue:
  ⬆(expected social values) – (expected <u>consumer's surplus</u>)⬇

  - Monopolist demand curve: $p_i = 100 - q_i$
  - Capacity: uniform distribution in $[0{,}100]$

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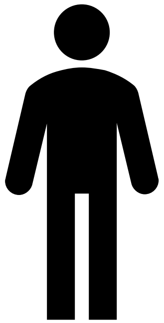

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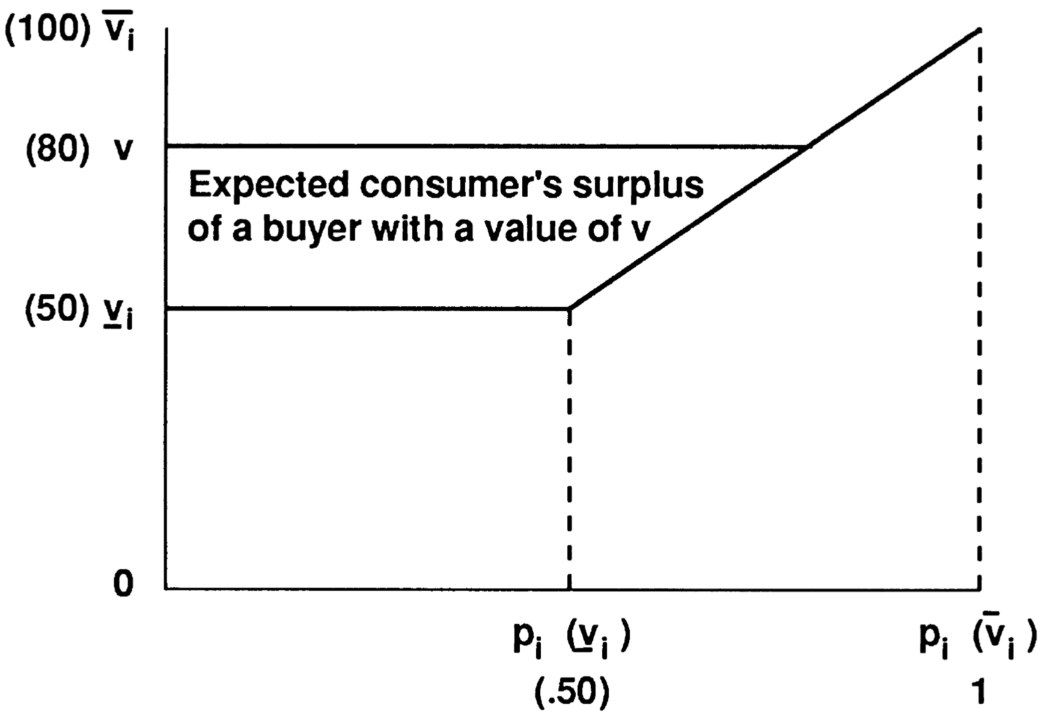
Buyer from market i



Price Discriminating Monopoly

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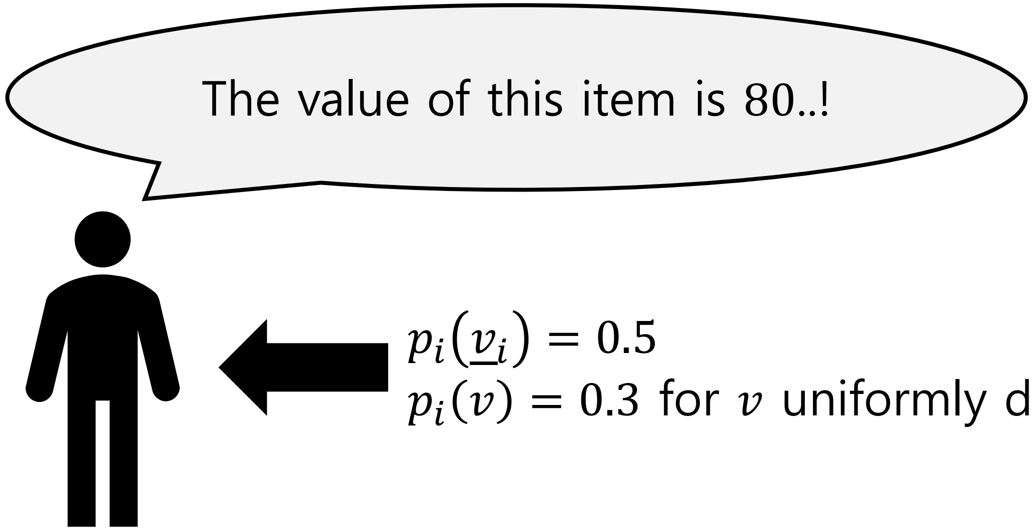
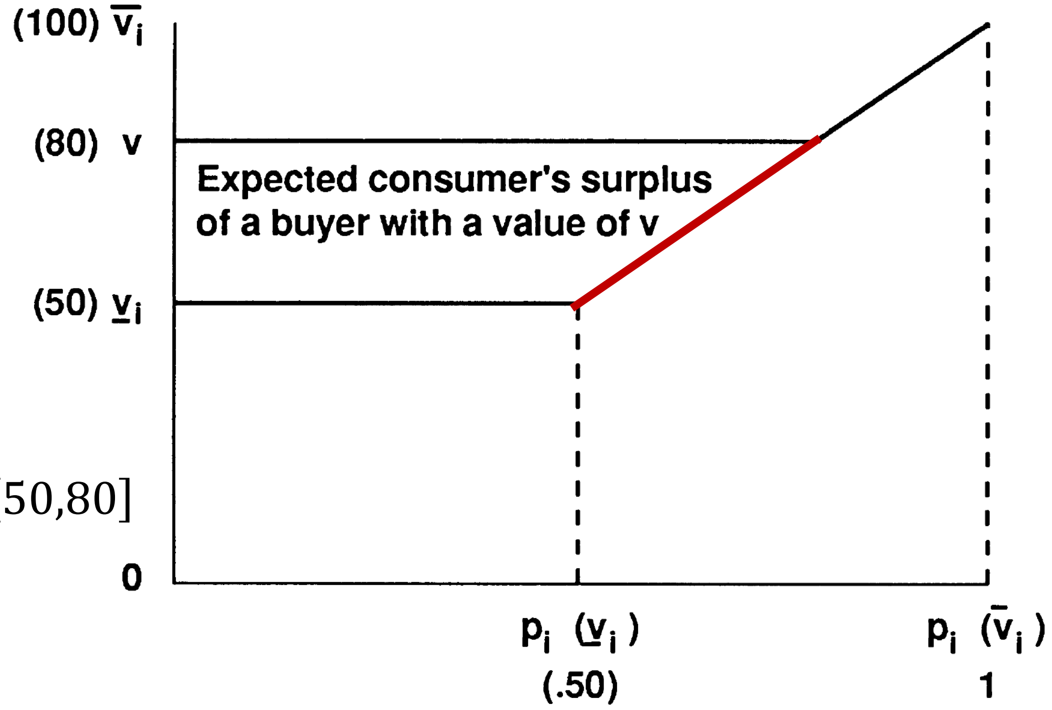


Buyer from market i

$p_i(\underline{v}_i) = 0.5$
$p_i(v) = 0.3$ for v uniformly distributed in $[50,80]$



Price Discriminating Monopoly

- Maximize monopolist's expected revenue:
  ⬆(expected social values) – (expected consumer's surplus)⬇

- Total expected consumer surplus

$$\int_{\underline{Q}}^{\overline{Q}} h(Q) \sum_{i=1}^{n} \int_{\underline{v}_i}^{\overline{v}_i} f_i(v) \int_{\underline{v}_i}^{v} p_i(x, Q) dx dv dQ$$

Price Discriminating Monopoly

- Maximize monopolist's expected revenue:
  ⬆(expected social values) – (expected consumer's surplus)⬇

- Total expected consumer surplus

$$\sum_{i=1}^{n} \int_{\underline{v}_i}^{\overline{v}_i} f_i(v) \int_{\underline{v}_i}^{v} \overline{p}_i(x) dx dv$$

Price Discriminating Monopoly

- Maximize monopolist's expected revenue:
  ⬆(expected social values) – (expected consumer's surplus)⬇

- Expected social values

$$\sum_{i=1}^{n}\int_{\underline{v}_i}^{\overline{v}_i} v f_i(v)\overline{p}_i(v)dv$$

- Total expected consumer surplus

$$\sum_{i=1}^{n}\int_{\underline{v}_i}^{\overline{v}_i} f_i(v)\int_{\underline{v}_i}^{v}\overline{p}_i(x)dxdv$$

Price Discriminating Monopoly

- Maximize monopolist's expected revenue:
  ⬆(expected social values) – (expected consumer's surplus)⬇

  subject to the constraint to capacity:

$$\sum_{i=1}^{n} \int_{\underline{v}_i}^{\overline{v}_i} f_i(v) p_i(v, Q) dv \leq Q$$

Price Discriminating Monopoly

- Maximize monopolist's expected revenue:
  ⬆(expected social values) – (expected consumer's surplus)⬇

$$\pi(Q) = \sum_{i=1}^{n} \int_{\underline{v}_i}^{\overline{v}_i} v f_i(v) p_i(v,Q)\,dv - \sum_{i=1}^{n} \int_{\underline{v}_i}^{\overline{v}_i} f_i(v) \int_{\underline{v}_i}^{v} p_i(x,Q)\,dx\,dv$$

subject to:

$$\sum_{i=1}^{n} \int_{\underline{v}_i}^{\overline{v}_i} f_i(v) p_i(v,Q)\,dv \le Q$$

Price Discriminating Monopoly

- Maximize monopolist's expected revenue:
  ⬆(expected social values) – (expected consumer's surplus)⬇

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subject to:

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Defines $z(v, x) = \begin{cases} 1 & x \leq v \\ 0 & x > v \end{cases}$

Price Discriminating Monopoly

- Maximize monopolist's expected revenue:
  ⬆(expected social values) – (expected consumer's surplus)⬇

$$\pi(Q) = \sum_{i=1}^{n} \int_{\underline{v}_i}^{\overline{v}_i} v f_i(v) p_i(v,Q) dv - \sum_{i=1}^{n} \int_{\underline{v}_i}^{\overline{v}_i} \int_{\underline{v}_i}^{\overline{v}_i} f_i(v) p_i(x,Q) z(v,x) dx dv$$

Defines $z(v,x) = \begin{cases} 1 & x \le v \\ 0 & x > v \end{cases}$

subject to:

$$\sum_{i=1}^{n} \int_{\underline{v}_i}^{\overline{v}_i} f_i(v) p_i(v,Q) dv \le Q$$

Price Discriminating Monopoly

- Maximize monopolist's expected revenue:
  ⬆(expected social values) – (expected consumer's surplus)⬇

$$\pi(Q) = \sum_{i=1}^{n} \int_{\underline{v}_i}^{\overline{v}_i} v f_i(v) p_i(v, Q) dv - \sum_{i=1}^{n} \int_{\underline{v}_i}^{\overline{v}_i} [1 - F_i(v)] p_i(v, Q) dv$$

subject to:

$$\sum_{i=1}^{n} \int_{\underline{v}_i}^{\overline{v}_i} f_i(v) p_i(v, Q) dv \leq Q$$

Price Discriminating Monopoly

- Maximize monopolist's expected revenue:
  ⬆(expected social values) – (expected consumer's surplus)⬇

$$ER = \sum_{i=1}^{n} \int_{\underline{v}_i}^{\overline{v}_i} \left[v - \frac{1 - F_i(v)}{f_i(v)} \right] f_i(v) p_i(v, Q) dv$$

subject to:

$$\sum_{i=1}^{n} \int_{\underline{v}_i}^{\overline{v}_i} f_i(v) p_i(v, Q) dv \leq Q$$

Price Discriminating Monopoly

- Maximize monopolist's expected revenue:
  ⬆(expected social values) – (expected consumer's surplus)⬇

$$ER = \sum_{i=1}^{n} \int_{\underline{v}_i}^{\overline{v}_i} MR_i(v) f_i(v) p_i(v, Q) dv$$

subject to:

$$\sum_{i=1}^{n} \int_{\underline{v}_i}^{\overline{v}_i} f_i(v) p_i(v, Q) dv \leq Q$$

Reinterpretation

- Recall both optimal auction and monopoly problem
  - No buyer/customer pay a price below $\underline{v}$
  - Payment with respect to the marginal revenue

- Assumptions in common
  - No resale
  - Risk neutrality

Reinterpretation

- Target
  - Maximize seller's expected revenue
  - Maximize monopolist's expected revenue

- Basic notations
  - n: number of bidders
  - n: number of markets
  - $F_i(v)$: prob. that buyer i's value $< v$
  - $F_i(v)$: number of buyers in market i with value $< v$

Reinterpretation

- Target
  - Maximize seller's expected revenue
  - Maximize monopolist's expected revenue
- Prob. that a buyer with value v in market i wins (in auction, ith bidder)
  - Function of the values of all the bidders: $E\left(p_i(v_1, \dots, v_{i-1}, v, v_{i+1}, \dots, v_n)\right)$
  - Function of his value and the quantity

Reinterpretation

- Target
  - Maximize seller's expected revenue
  - Maximize monopolist's expected revenue

- Prob. that ith bidder with value v wins
  - $\overline{p}_i(v) \coloneqq E\left(p_i(v_1, \dots, v_{i-1}, v, v_{i+1}, \dots, v_n)\right)$

  → Unconditional prob. that the bidder with distribution F_i receives the good

Reinterpretation

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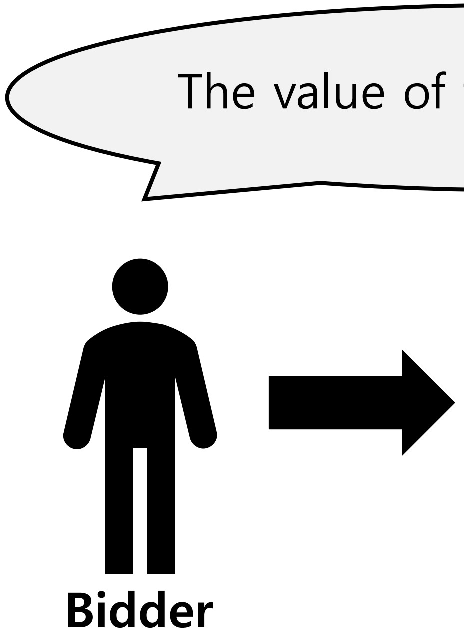
  → Unconditional prob. that the bidder with distribution F_i receives the good



Achieve surplus: (surplus from v) + $\overline{p}_i(v)dv$

Increasing at a rate of $\overline{p}_i(v)$

Reinterpretation

- Prob. that ith bidder with value v wins
  - $\overline{p}_i(v) := E\left(p_i(v_1, \ldots, v_{i-1}, v, v_{i+1}, \ldots, v_n)\right)$

  → Unconditional prob. that the bidder with distribution F_i receives the good

- Define $S_i(v)$ as the expected surplus of the bidder i with value v

$$\frac{\partial S_i(v)}{\partial v} = \overline{p}_i(v) \geq 0$$

Reinterpretation

- Prob. that ith bidder with value v wins
  - $\overline{p}_i(v) \coloneqq E\left(p_i(v_1, \dots, v_{i-1}, v, v_{i+1}, \dots, v_n)\right)$
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- Prob. that ith bidder with value v wins
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  → Unconditional prob. that the bidder with distribution F_i receives the good

- Define $S_i(v)$ as the expected surplus of the bidder i with value v

$$\int_{\underline{v}_i}^{v} \overline{p}_i(x)dx$$

Reinterpretation

- Prob. that ith bidder with value v wins
  - $\overline{p}_i(v) \coloneqq E\left(p_i(v_1, \dots, v_{i-1}, v, v_{i+1}, \dots, v_n)\right)$

  → Unconditional prob. that the bidder with distribution F_i receives the good

- Total expected surplus of all the bidders

$$\sum_{i=1}^{n} \int_{\underline{v}_i}^{\overline{v}_i} f_i(v) \int_{\underline{v}_i}^{v} \overline{p}_i(x) dx dv$$

…surprisingly, the same with the monopoly problem!!

Reinterpretation

Maximize

$$\sum_{i=1}^{n}\int_{\underline{v}_i}^{\overline{v}_i} v f_i(v)\overline{p}_i(v)dv - \sum_{i=1}^{n}\int_{\underline{v}_i}^{\overline{v}_i} f_i(v)\int_{\underline{v}_i}^{v}\overline{p}_i(x)dxdv$$

subject to

$$\sum_{i=1}^{n} p_i(v_1, \dots, v_n) \leq 1$$

Reinterpretation

Maximize

$$\sum_{i=1}^{n}\int_{\underline{v}_i}^{\overline{v}_i} v f_i(v)\overline{p}_i(v)dv - \sum_{i=1}^{n}\int_{\underline{v}_i}^{\overline{v}_i} f_i(v)\int_{\underline{v}_i}^{v}\overline{p}_i(x)dxdv$$

subject to

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Reinterpretation

Maximize

$$\sum_{i=1}^{n} \int_{\underline{v}_i}^{\overline{v}_i} v f_i(v) \overline{p}_i(v) dv - \sum_{i=1}^{n} \int_{\underline{v}_i}^{\overline{v}_i} f_i(v) \int_{\underline{v}_i}^{v} \overline{p}_i(x) dx dv$$

subject to

$$\sum_{i=1}^{n} p_i(v_1, \dots, v_n) \leq 1$$

Reinterpretation

Maximize

$$ER = \sum_{i=1}^{n} \int_{\underline{v}_i}^{\overline{v}_i} MR_i(v) f_i(v) \overline{p}_i(v) dv \ - K$$

subject to

$$\sum_{i=1}^{n} p_i(v_1, \dots, v_n) \leq 1$$

Reinterpretation

Maximize

$$ER = \text{expected MR of winning bidder } - K$$

subject to

$$\sum_{i=1}^{n} p_i(v_1, \dots, v_n) \leq 1$$

Reinterpretation

Maximize

Consumer surplus

$$ER = \text{expected MR of winning bidder } - K$$

subject to

$$\sum_{i=1}^{n} p_i(v_1, \dots, v_n) \leq 1$$

Reinterpretation

Maximize

$$ER = \text{expected MR of winning bidder } - 0$$

subject to

$$\sum_{i=1}^{n} p_i(v_1, \dots, v_n) \leq 1$$

→ Optimal auction

Reinterpretation

Maximize

$$ER = \text{expected MR of winning bidder } - 0$$

subject to

$$\sum_{i=1}^{n} p_i(v_1, \dots, v_n) \leq 1$$

→ Optimal auction

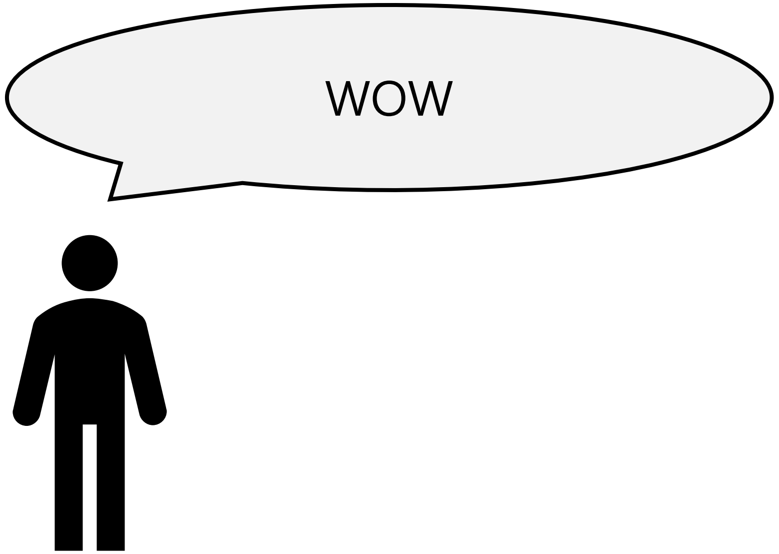
Reinterpretation

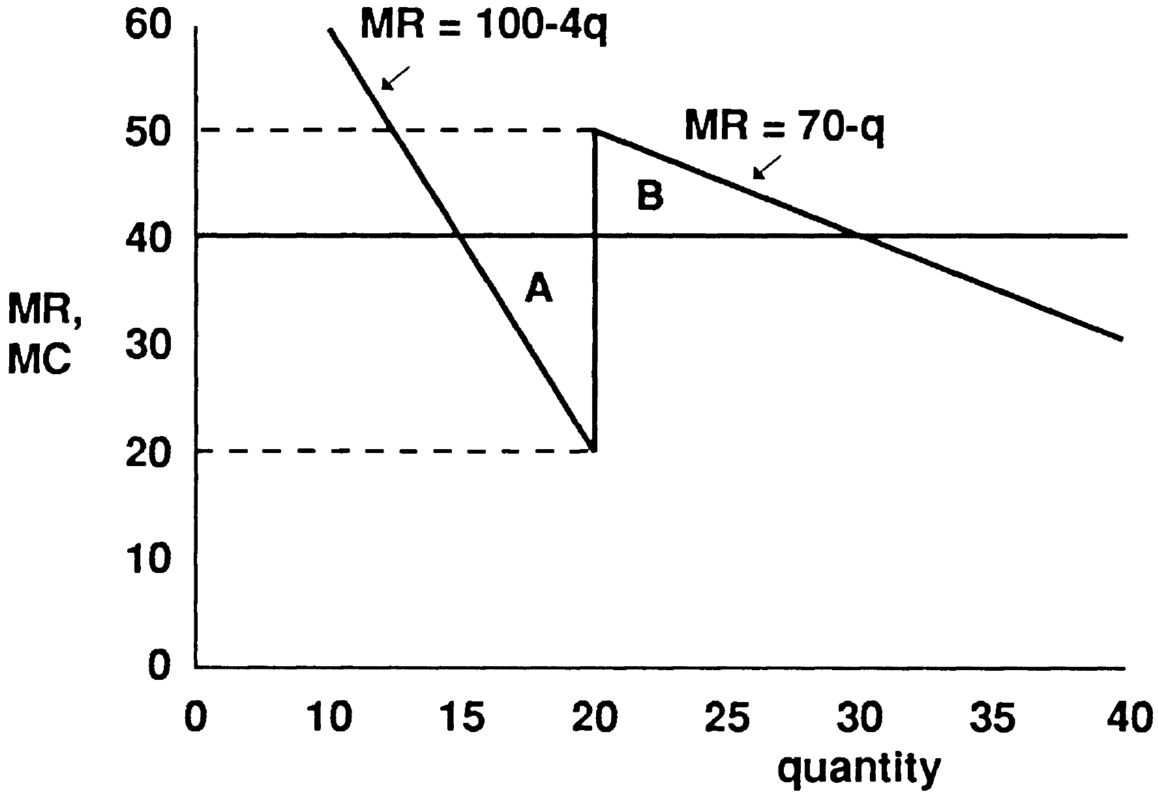
- Objective function of maximizing expected marginal revenue

- Optimize similarly
  - Unit allocation with priority to marginal revenue
  - Allocate until 1) quantity is exhausted or 2) no non-negative marginal revenue

Some remains…

What if marginal revenues are not downward sloping?





Some remains…

When buyers’ and the seller’s valuation are private

Thank you

Thank you for listening.
Tell me if you have any questions