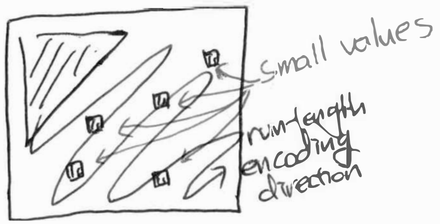
Further compression?



After Quantization

We want to discard "small" values to have shorter run-length encoding w/o too much distortion.

⇒ Prob Given an 8x8 ~~img~~ coeff. matrix $\hat{X}$ which is the quantization of coeff. matrix X, minimize

distortion $$D(X, \tilde{X} \mid \hat{X})$$

over all thresholded version of $\hat{X}$, subject to $\tilde{X}$, the

rate $$R(\tilde{X}) \le R_{budget}.$$

As always, use the Lagrange multiplier:

$$\min J(\lambda) = \min \left[D(X, \tilde{X}) + \lambda R(\tilde{X}) \right].$$

Define

$$J_{min}(\lambda) = \min_{\tilde{X}} J(\lambda)$$

Then,

$$J_{min}(\lambda^*) = \max_{\lambda \ge 0} \left[J_{min}(\lambda) - \lambda R_{budget} \right]$$

Alg $J_{min}(\lambda)$

Given $X, \hat{X}$ and λ, s.t. $\hat{X}_i, \hat{X}_j$ is nonzero, $i < j$.

FOR $i, j \in \{0, 1, \dots, 63\}$ do — change in mse

  $E_k \leftarrow X_k^2 - (X_i - \hat{X}_i)^2$

  $R_{i,j} \leftarrow$ # additional bits required to encode jth coeff, conditioned on the case where the ith coeff is the prev non zero coeff. — change in bit rate

end FOR $k^* \leftarrow 0$

FOR $j \in \{0, 1, \dots, 63\}$ do

  FOR $i \in \{0, 1, \dots, j-1\}$ do

    $\Delta J_{i,j} \leftarrow -E_j + \lambda R_{i,j}$

  end FOR.

  $J_j^* = \min_{i \in \{0, \dots, j-1\}} \left[J_i^* + \Delta J_{i,j} \right]$

  if $J_k^* \le J_{k^*}^*$, $k^* \leftarrow k$.

end FOR

Backtrack for the optimal sequences that are not "cut"

return $\sum_{(\text{backtrack seq})} \Delta J_{i,j}$

Thm We can find λ^* and the opt $\hat{X}$ w.r.t. Rate-Distortion using $J_{min}(\lambda)$ and

$$J_{min}(\lambda^*) = \max_{\lambda \geq 0} \left[J_{min}(\lambda) - \lambda R_{budget} \right]$$

using a bisection algorithm.

- Kannan Ramchandran and Martin Vetterli, "Rate-Distortion Optimal Fast Thresholding with Complete JPEG/MPEG Decoder Compatibility." IEEE Transactions on image processing '94.